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Structured H_∞ Command and Control-Loop Design for Unmanned Helicopters

J. Gadewadikar*
Alcorn State University, Lorman Mississippi 39096
F. L. Lewis† and Kamesh Subbarao‡
University of Texas at Arlington, Fort Worth, Texas 76118
and
Ben M. Chen§
National University of Singapore, Singapore 117576, Republic of Singapore

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The aim of this paper is to present rigorous and efficient methods for designing flight controllers for unmanned helicopters that have guaranteed performance, intuitive appeal for the flight control engineer, and prescribed multivariable loop structures. Helicopter dynamics do not decouple as they do for the fixed-wing aircraft case, and so the design of helicopter flight controllers with a desirable and intuitive structure is not straightforward. We use an H_∞ output-feedback design procedure that is simplified in the sense that rigorous controller designs are obtained by solving only two coupled-matrix design equations. An efficient algorithm is given for solving these that does not require initial stabilizing gains. An output-feedback approach is given that allows one to selectively close prescribed multivariable feedback loops using a reduced set of the states at each step. At each step, shaping filters may be added that improve performance and yield guaranteed robustness and speed of response. The net result yields an H_∞ design with a control structure that has been historically accepted in the flight control community. As an example, a design for stationkeeping and hover of an unmanned helicopter is presented. The result is a stationkeeping hover controller with robust performance in the presence of disturbances (including wind gusts), excellent decoupling, and good speed of response.

Nomenclature

A	=	system or plant matrix
a_s	=	longitudinal blade angle
B	=	control-input matrix
b_s	=	lateral blade angle
C	=	output or measurement matrix
D	=	disturbance matrix
D_{in}	=	inner-loop disturbance matrix
D_o	=	outer-loop disturbance matrix
$d(t)$	=	disturbance
G	=	nominal plant
G_s	=	loop-shaped plant
K	=	static output-feedback gain matrix
p	=	roll rate in the body-frame components
Q	=	state weighting matrix
q	=	pitch rate in the body-frame components
R	=	control weighting matrix
r	=	yaw rate in the body-frame components
r_{fb}	=	yaw-rate feedback
U	=	velocity along the body-frame x axis
$u(t)$	=	control input
V	=	velocity along the body-frame y axis
W	=	velocity along the body-frame z axis
X	=	inertial position x axis
$x_{\mathrm{in}}(t)$	=	inner-loop state vector
$x_o(t)$	=	outer-loop state vector
Y	=	inertial position y axis
$y_{\mathrm{in}}(t)$	=	inner-loop output vector
$y_o(t)$	=	outer-loop output vector
Z	=	inertial position z axis
$z(t)$	=	performance output
γ	=	system L_2 gain
γ_{in}	=	L_2 gain inner loop
γ_o	=	L_2 gain outer loop
θ	=	pitch angle
ϕ	=	roll angle
ψ	=	yaw angle

I. Introduction

OVER the past few years, there has been significant interest in using unmanned aerial vehicles for applications such as search and rescue, surveillance, and remote inspection. Rotorcraft (especially helicopters) have several significant advantages over conventional fixed-wing platforms in conducting several of these tasks. The advantages are exemplified by certain unique capabilities of rotorcraft; for example, they can hover and they can take off and land in very limited spaces. The ability to reliably follow prescribed 3-D position and yaw commands in the presence of disturbances is a requirement common to rotary-wing unmanned aerial vehicles (UAVs). Position-tracking control system design for a UAV is challenging because strong coupling among all states is present in the rotorcraft, which must be confronted in any design technique. Moreover, the rotor flexibility dynamics must generally be included in any design to guarantee stability robustness [1]. In fixed-wing aircraft control, by contrast, the dynamics conveniently decouple in

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*Assistant Professor, Robotics and Automation, Department of Advanced Technologies, 1000 ASU Drive, No. 360; jyo@alcorn.edu.

†Associate Director of Research, Automation and Robotics Research Institute, 7300 Jack Newell Boulevard South; Professor, Department of Electrical Engineering.

‡Assistant Professor, Department of Mechanical and Aerospace Engineering, 500 West First Street, Box 19018, 211 Woolf Hall. Life Member AIAA.

§Professor, Department of Electrical and Computer Engineering.

equilibrium flight. There is a body of knowledge and expertise in the design of flight controllers that have suitable structure for specific applications [2]. Multiloop controllers can be designed with inner rate loops and outer position- or attitude-control loops. In helicopter control, the dynamics do not decouple and all the dynamics must generally be used when closing any loops. This makes it difficult to provide rotorcraft flight controllers with desirable intuitive structures.

The aim of this paper is to present rigorous and efficient methods for designing flight controllers for unmanned helicopters that have guaranteed performance, intuitive appeal for the flight controls engineer, and prescribed multivariable loop structures. We use an H_∞ output-feedback design procedure that allows one to selectively close prescribed multivariable feedback loops and to preserve control structures that have been historically accepted in the flight control community, while obtaining guaranteed performance.

Several linear and nonlinear control strategies have been proposed for control of helicopters and samples can be found in [3–5], such as the adaptive feedback linearization approach, in which a neural network approximates the uncertainties and the network weights are updated adaptively, based on the trajectory-tracking errors [3,4]. Although the controllers are efficient, one introduces additional dynamics to synthesize the controller. Further, this makes the controller of very high order and there is no optimality guaranteed against the specific classes of disturbances/uncertainties considered.

Multiloop approaches are discussed extensively in [6–8]. A static inner loop was computed using eigenstructure assignment, and an outer dynamic loop was computed using H_∞ synthesis [6]. An inner loop was designed using the linear quadratic regulator (LQR) technique, a standard H_∞ outer loop [7], and four body angular measurements (roll and pitch angles and rates), and the rotor lag and flap state measurements and their derivatives are fed back. An inner-loop attitude feedback control, midloop velocity feedback control, and the outer-loop position-control approach on an UAV is shown in [8], in which a multiloop single-input/single-output control structure is employed. Other approaches include helicopter control based on eigenstructure assignment using all the states simultaneously [9] and a hybrid μ/H_∞ design and its application to flight control of a helicopter [10]. To summarize, a lot of work has been pursued, but several problems are still open. Many of the approaches are theoretically appealing but have one or more of the following issues: they are difficult to implement, they lack structure, they do not have disturbance rejection formulation, they are difficult to solve for higher-order systems, and they can impose numerical problems.

In this paper, we employ a static output-feedback (OPFB) control structure based on H_∞ theory. The static output-feedback approach allows one to selectively close prescribed multivariable feedback loops and preserve control structures that have been historically accepted in the flight controls community, while obtaining guaranteed performance. Static OPFB design, as opposed to dynamic output feedback with a regulator, is suitable for the design of aircraft controllers of prescribed structure [2].

It is well known that the OPFB optimal control solution can be prescribed in terms of three coupled-matrix equations [11]: namely, two associated Riccati equations and a spectral radius coupling equation. A sequential numerical algorithm to solve these equations is presented in [12]. OPFB stabilizability conditions that only require the solution of two coupled-matrix equations are given in [13–15]. Linear quadratic suboptimal control with static output feedback involving linear matrix inequalities (LMIs) is discussed [16]. Some recent LMI approaches for OPFB design are presented in [17–19]. These allow the design of OPFB controllers using numerically efficient software (e.g., LMI Control Toolbox for MATLAB [20]). Many of the solution algorithms are difficult to implement, difficult to solve for higher-order systems, may impose numerical problems, and may have restricted solution procedures such as the initial stabilizing-gain requirements.

H_∞ design has been considered for static OPFB; Hol and Scherer [21] addressed the applicability of a matrix-valued sum-of-squares techniques for the computations of LMI lower bounds. Conditions for a static output loop-shaping controller in terms of two coupled-matrix inequalities are presented in [22]. The application of loop-shaping procedures has lead to several improvements in helicopter control system design methods [23].

In this paper, we demonstrate that high-performance low-order controllers for the robust stabilization of autonomous helicopters can be easily and efficiently computed using the H_∞ static output-feedback techniques given in [4,24,25]. We present an approach to helicopter flight control design based on output-feedback H_∞ techniques that allows one to selectively close prescribed multivariable feedback loops one loop at a time, using a reduced set of states for feedback at each step. At each step, shaping filters may be added to improve performance and provide guaranteed robustness and good speed of response. The output-feedback approach allows one to selectively close prescribed multivariable feedback loops using a reduced set of states and to preserve control structures that have been historically accepted in the flight controls community, while obtaining guaranteed performance.

We use an H_∞ approach for static OPFB design that only requires the solution of one associated Riccati equation and a coupled-gain-matrix condition. This is more straightforward than optimal control techniques, which generally require solving three coupled-matrix equations. An efficient recursive algorithm is given for solving the output-feedback H_∞ design problem that does not require initial stabilizing gains.

As an example, we present a design for stationkeeping and hover control of an unmanned helicopter. The hover configuration in general is an unstable configuration, which can be effectively modeled using a linear model. In the presence of disturbances, the helicopter exhibits deviations in the dynamical states that complicate the control problem because the helicopter dynamical states are very tightly coupled. For example, in hover, pitch motion is almost always accompanied by forward and vertical motion and all three states need to be controlled simultaneously [26]. Moreover, the rotor flexible dynamics must be included [1].

An inner loop is first designed for attitude control (roll and pitch angles and all three body rates) using a subset of the state variables by output-feedback design. In the next formulation, shaping filters are added and a second output-feedback design using a second subset of the states is closed to provide position (X, Y, Z) and yaw tracking. The result is a stationkeeping hover controller with robust performance in the presence of disturbances (including wind gusts), excellent decoupling, and good speed of response, as verified by simulation. Though the net result is a controller that does feed back most of the state variables (though not the rotor flexible states), a two-loop design is used in which we use only a reduced set of states in each loop for feedback. This gives better structure and robustness. The net result is that the inner attitude-control loop is faster than the outer position-control loop, so that the attitudes respond quickly to allow effective position-tracking.

The paper is organized as follows. Section II details the formulation of necessary and sufficient conditions for H_∞ OPFB control. A solution algorithm is proposed in Sec. III. Section IV illustrates the UAV model controller structure, H_∞ loop-shaping design procedure, and simulation results with disturbance effects.

II. Necessary and Sufficient Conditions for H_∞ OPFB Control

In this section, we present a method for finding H_∞ static OPFB gains. It is seen that the H_∞ OPFB gain is computed in terms of only two coupled-matrix equations. This is a simpler problem to solve than the optimal OPFB problem given in terms of three coupled equations [11]. Moreover, a numerical algorithm is given to solve these equations that does not require an initial stabilizing OPFB gain.

A. System Description and Definitions

Consider the linear time-invariant system of Fig. 1 with control input $u(t)$, output $y(t)$, and disturbance $d(t)$ given by

$$\dot{x} = Ax + Bu + Dd, \qquad y = Cx \tag{1}$$

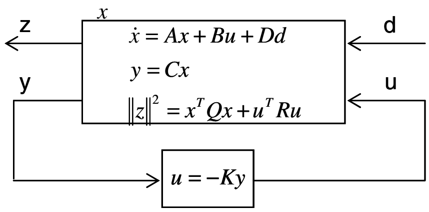

Fig. 1 System description.

and a performance output $z(t)$ that satisfies

$$\|z(t)\|^2 = x^T Q x + u^T R u \tag{2}$$

with $Q^T = Q \geq 0$ and $R^T = R > 0$. It is assumed that C has full row rank (a standard assumption to avoid redundant measurements).

A static output-feedback control is given by

$$u = -Ky = -KCx \tag{3}$$

By definition, the pair (A, B) is said to be *stabilizable* if there exists a real matrix K such that $A - BK$ is (asymptotically) stable. The pair (A, C) is said to be *detectable* if there exists a real matrix L such that $A - LC$ is stable. System (1) is said to be *output-feedback stabilizable* if there exists a real matrix K such that $A - BKC$ is stable.

B. Bounded L_2 Gain Design Problem

The system L_2 gain is said to be *bounded or attenuated by* γ if

$$\frac{\int_0^\infty \|z(t)\|^2 \, \mathrm{d}t}{\int_0^\infty \|d(t)\|^2 \, \mathrm{d}t} = \frac{\int_0^\infty (x^T Q x + u^T R u) \, \mathrm{d}t}{\int_0^\infty (d^T d) \, \mathrm{d}t} \leq \gamma^2 \tag{4}$$

for any nonzero energy-bounded disturbance input d. Call γ^* the minimum gain for which this occurs. For linear systems, there are explicit formulas to compute the minimum gain [27]. Throughout this paper, we shall assume that γ is fixed and $\gamma > \gamma^*$. The case when $\gamma = \gamma^*$ is called H_∞ control. It is desired to find a static OPFB gain K such that the system is stable and the L_2 gain is bounded by a prescribed value γ. The actuator signal is mathematically represented in the L_2 gain definition. This approach allows the adjustment of both for the best L_2 gain performance and a LQR-type relative weighing [11].

The following theorem gives necessary and sufficient conditions for the existence of bounded L_2 gain static OPFB control [25].

Theorem: For a given $\gamma > \gamma^*$, there exists an OPFB gain such that $A_0 \equiv (A - BKC)$ is asymptotically stable, with L_2 gain bounded by γ if and only if

1) (A, C) is detectable and there exist matrices L and $P = P^T \geq 0$ such that

2)

$$KC = R^{-1}(B^T P + L) \tag{5}$$

3)

$$PA + A^T P + C^T C + \frac{1}{\gamma^2} PDD^T P - PBR^{-1}B^T P + L^T R^{-1} L = 0 \tag{6}$$

III. Solution Algorithm

Most existing iterative algorithms for OPFB design require the determination of an initial stabilizing gain, which can be very difficult for practical aerospace systems such as the stabilization of an autonomous rotorcraft in hover. The following algorithm is proposed to solve the two coupled design equations in the preceding Theorem. Note that it does not require an initial stabilizing gain, as opposed to Kleinman's [28] state-feedback algorithm and the OPFB algorithm of Moerder and Calise [12], because it uses a Riccati equation solution, not a Lyapunov equation, at each step.

1) Initialize: set $n = 0$ and $L_0 = 0$, and select γ, Q, and R.

2) nth iteration: solve for P_n in

$$P_n A + A^T P_n + Q + \frac{1}{\gamma^2} P_n DD^T P_n - P_n BR^{-1} B^T P_n + L_n^T R^{-1} L_n = 0 \tag{7}$$

Evaluate gain and update L

$$K_{n+1} = R^{-1}(B^T P_n + L_n) C^T (CC^T)^{-1} \tag{8}$$

$$L_{n+1} = RK_{n+1} C - B^T P_n \tag{9}$$

Check convergence. If converged, go to step 3, otherwise set $n = n + 1$ and go to step 2.

3) Terminate: set $K = K_{n+1}$.

The convergence can be checked using the norm of $K_{n+1} - K_n$ (e.g., $\|K_{n+1} - K_n\| < \varepsilon$, where ε is a small number and operator $\|\ \|$ denotes the matrix norm).

Note that this algorithm uses well-developed techniques for solving available Riccati equations, for instance, those provided in MATLAB. It generalizes the algorithm in [15] to the case of nonzero initial gain. It is shown in Sec. IV that this algorithm is also suitable to find static output-feedback gains for loop-shaped plants.

Lemma: If this algorithm converges, it provides the solution to Eqs. (5) and (6).

Proof: Clearly, at convergence, Eq. (7) holds for P_n. Note that substitution of Eq. (8) into Eq. (9) yields

$$L_{n+1} = R[R^{-1}(B^T P_n + L_n) C^+] C - B^T P_n$$

At convergence, $L_{n+1} = L_n \equiv L$ and $P_n \equiv P$, so that

$$L = (B^T P + L) C^+ C - B^T P$$

or

$$B^T P + L = (B^T P + L) C^+ C$$

This guarantees that there exists a solution K to Eq. (5) given by $K = R^{-1}(B^T P + L) C^+$. □

IV. Design Example

The H_∞ output feedback is illustrated via a UAV simulation. First, an inner loop is designed for attitude control (roll and pitch angles and all three body rates) using a subset of the state variables by output-feedback design. Then shaping filters are added and a second output-feedback design is demonstrated using a second subset of the states to provide position (X, Y, Z) and yaw tracking.

A. System Description

The controller design is based on an 11-state linear model of a Raptor-90 helicopter, shown in Fig. 2. The results are based on the

Fig. 2 Raptor-90 helicopter.

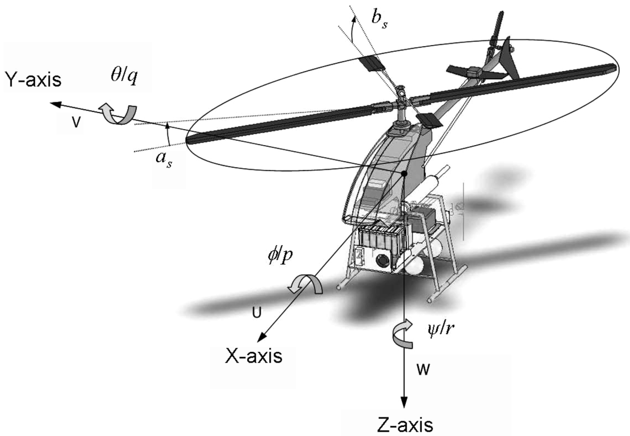

Fig. 3 Helicopter states in body-frame coordinate system.

model derived at the National University of Singapore. A linearized model for the hover operating condition was established. The model currently used is a state-space representation of a helicopter, modeled as a six-degree-of-freedom rigid-body augmented with servo/rotor dynamics and artificial yaw-damping dynamics [29]. The state vector physically shown in Fig. 3 contains 11 states and can be expressed as

$$x_{\mathrm{in}} = [\,U \quad V \quad p \quad q \quad \phi \quad \theta \quad a_s \quad b_s \quad W \quad r \quad r_{\mathrm{fb}}\,]^T$$

The yaw-rate feedback r_{fb} is included in the state vector. For miniature rotorcraft, r_{fb} could be expressed by a first-order low-pass filter [30]. The rotor flexibility dynamics a_s, b_s are also included, as required for robust controller design [1]. These states cannot be used for feedback purposes, immediately rendering state-feedback design methods inappropriate for these states.

The input vector can be written as $u_{\mathrm{in}} = [\,\delta_{\mathrm{lat}} \quad \delta_{\mathrm{long}} \quad \delta_{\mathrm{col}} \quad \delta_{\mathrm{ped}}\,]^T$, where δ_{lat} is the lateral channel input and affects roll motion, δ_{long} is longitudinal channel input and affects pitch, δ_{ped} is pedal channel input that affects yaw motion, and δ_{col} is the collective channel. In helicopters, there is a high degree of coupling between lateral and longitudinal dynamics [5].

B. Inner-Loop Controller Design

In a recent publication by Gadewadikar et al. [31], loop-shaping and H_∞ methods are applied to track pitch and roll. The objective of this paper is to offer position control, and hence an inner loop is first designed that stabilizes the orientation; that is, the primary variables to be controlled in the inner loop are the pitch angle, the roll angle, and three extra rate gyros measuring pitch angular rate, roll angular rate, and yaw angular rate will also be used for feedback purposes. Thus, five system states constitute the output vector for the inner loop $y_{\mathrm{in}} = [\,\phi \quad \theta \quad p \quad q \quad r\,]^T$. The rotorcraft equations mentioned were trimmed in a hover configuration to obtain the reference trim condition. The nonlinear equations were then linearized for the hover configuration based on the reference values obtained [2]. The state variable model of the helicopter is of the form

$$\begin{gathered} \dot{x}_{\mathrm{in}} = A_{\mathrm{in}} x_{\mathrm{in}} + B_{\mathrm{in}} u_{\mathrm{in}} + D_{\mathrm{in}} d_{\mathrm{in}}, \qquad y_{\mathrm{in}} = C_{\mathrm{in}} x_{\mathrm{in}} \\ x_{\mathrm{in}} \in R^{11}, \qquad u_{\mathrm{in}} \in R^4, \qquad y_{\mathrm{in}} \in R^5 \end{gathered} \tag{10}$$

where

$$A_{\mathrm{in}} = \begin{bmatrix} -0.1778 & 0 & 0 & 0 & 0 & -9.7807 & -9.7807 & 0 & 0 & 0 & 0 \\ 0 & -0.3104 & 0 & 0 & 9.7807 & 0 & 0 & 9.7807 & 0 & 0 & 0 \\ -0.3326 & -0.5353 & 0 & 0 & 0 & 0 & 75.7640 & 343.8600 & 0 & 0 & 0 \\ 0.1903 & -0.2940 & 0 & 0 & 0 & 0 & 172.6200 & -59.9580 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -8.1222 & 4.6535 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -0.0921 & -8.1222 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 17.1680 & 7.1018 & -0.6821 & -0.1070 & 0 \\ 0 & 0 & -0.2834 & 0 & 0 & 0 & 0 & 0 & -0.1446 & -5.5561 & -36.6740 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.7492 & -11.1120 \end{bmatrix}$$

$$B_{\mathrm{in}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.0632 & 3.3390 & 0 & 0 \\ 3.1739 & 0.2216 & 0 & 0 \\ 0 & 0 & 19.9250 & 0 \\ 0 & 0 & 2.0816 & -74.3640 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad C_{\mathrm{in}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and the disturbance dynamics D_{in} are discussed in Sec. IV.C.

The control structure shown in Fig. 4 is an attitude-stabilization control loop. In the presence of disturbances, the inner loop arrests the buildup in the rotational velocities and maintains the attitude of the helicopter by controlling the attitude states' roll and pitch. This is imperative for the hover controller (designed later), because deviations in pitch and roll angles will also result in X and Y translations.

An output-feedback control input of the form is selected as

$$u_{\mathrm{in}} = -K_{\mathrm{in}} y_{\mathrm{in}} + u_{\mathrm{ic}} \tag{11}$$

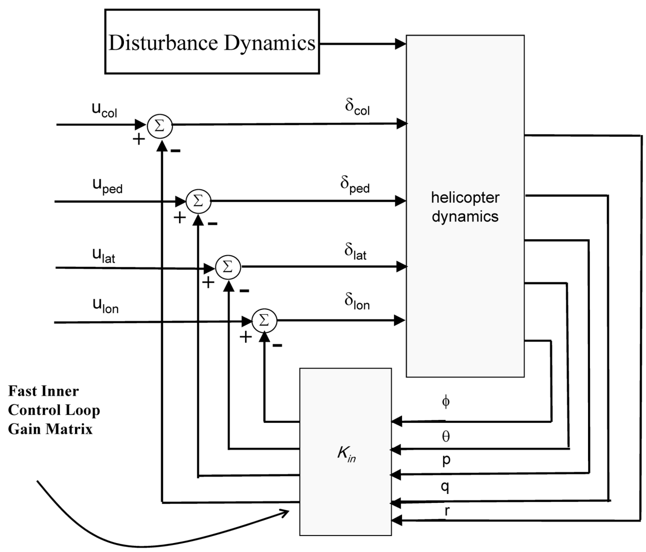

Fig. 4 Controller structure.

The input vector $u_{\mathrm{ic}} = [\,u_{\mathrm{lat}} \quad u_{\mathrm{long}} \quad u_{\mathrm{col}} \quad u_{\mathrm{ped}}\,]^T$ will be used as control input in the outer-loop design stage. The output-feedback gain K_{in} is determined from the algorithm described in Sec. III using the state variable model in Eq. (10).

For the computation of the output-feedback gain K_{in}, it is necessary to select weighting matrices Q and R. Note that in the inner-loop design Q is a constant (11×11) matrix and R is a constant (4×4) matrix, such that $Q^T = Q \geq 0$ and $R^T = R > 0$. A diagonal structure is used for Q and R. The diagonal entries are tuned iteratively; that is, for a given selection of Q and R, the algorithm is used to find the OPFB gain K_{in}. The closed-loop system is then simulated and if the results are not satisfactory, Q and R are modified and the procedure is repeated. To avoid the excitation of unmodeled high-frequency dynamics, the control input and velocity states are heavily penalized [30]. Note that the term $u^T Ru$ in Eq. (4) weights the deviations from the trim value of control, thereby reducing excessive control activity, which should be small to make the helicopter ride smoother and use less fuel. The resulting output-feedback gain K_{in} is

$$K_{\mathrm{in}} = \begin{bmatrix} 0.8984 & -0.0525 & 0.0306 & -0.0131 & 0 \\ 0.0445 & 0.9612 & 0.0084 & 0.0589 & 0 \\ -0.1648 & -0.2251 & -0.0626 & -0.0836 & -0.0037 \\ -0.0175 & -0.0249 & -0.0050 & -0.0097 & -0.2300 \end{bmatrix}$$

The gain parameter γ defines the desired L_2 gain bound. For the initial design, a fairly large γ is selected. If the algorithm converges, the parameter γ may be reduced. If γ is taken too small, the algorithm will not converge because the algebraic Riccati equation has no positive semidefinite solution. After some design repetitions, which were performed very quickly using the algorithm, we found the smallest value of the gain to be 0.62. To compare this with popular design methods [27], it was shown that the smallest value of L_2 gain was greater than that for state feedback and dynamic output feedback [25]. H_∞ static output-feedback methods have a bigger bound than state-feedback and dynamic output-feedback compensator methods, because the static output feedback is a subset of state feedback.

C. Wind Turbulence Model

The disturbance vector d given in Eq. (10) has wind components along the $[X \quad Y]^T$ fuselage axes, and disturbance input matrix D defines the dynamics involved with body-frame X and Y velocities. For this example, D_{in} is a 11×2 matrix and is constituted from the first two columns of the plant matrix A_{in}:

$$d_{\mathrm{in}} = [\,d_U \quad d_V\,]^T \tag{12}$$

Hall, Jr. and Bryson, Jr. [1] modeled the wind components along the fuselage axes by independently excited correlated Gauss–Markov processes:

$$\begin{bmatrix} \dot{d}_U \\ \dot{d}_V \end{bmatrix} = \begin{bmatrix} -1/\tau_c & 0 \\ 0 & -1/\tau_c \end{bmatrix} \begin{bmatrix} d_U \\ d_V \end{bmatrix} + \rho^* B_w \begin{bmatrix} q_U \\ q_V \end{bmatrix} \tag{13}$$

Equation (13) is a shaping filter for the wind, where q_U and q_V are independent with zero mean, $\tau_c = 3.2$ s is the correlation time of the wind σ_{q_U} ($\sigma_{q_V} = 20$ ft/s), B_w is the turbulence input identity matrix, and $\rho = 1/2$ is the scalar weighting factor.

D. Inner-Loop Simulation Results with Disturbance Effects

The static output-feedback solution derived in Sec. III is applied to obtain an output-feedback controller to stabilize a loop-shaped plant. The controller is then simulated, subject to the wind disturbances, to evaluate the efficacy of the proposed control law. The closed-loop system is shown in Fig. 5, in which the exogenous disturbance input $d(t)$ is a random variable, shown in Fig. 6, generated in the time domain to match statistical properties of the turbulence model, as discussed in Sec. IV.B.

To verify the design, a simulation is performed. The perturbed initial state was selected as

$$x_{\mathrm{in}}(0) = [\,0 \quad 0 \quad 0 \quad 0 \quad 0.1 \quad 0.2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\,]^T$$

The closed-loop responses in roll and pitch are given in Fig. 7. It can be observed that roll and pitch angles converge very quickly, indicating a fast inner loop. It is important to mention that although the inner loop has good control on the attitude states, it cannot hold

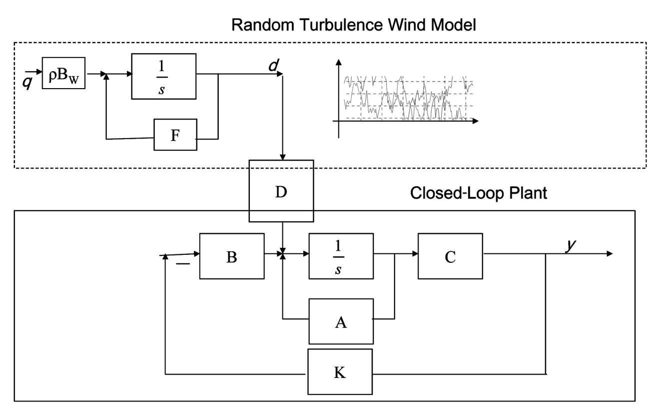

Fig. 5 Simulation with turbulence model.

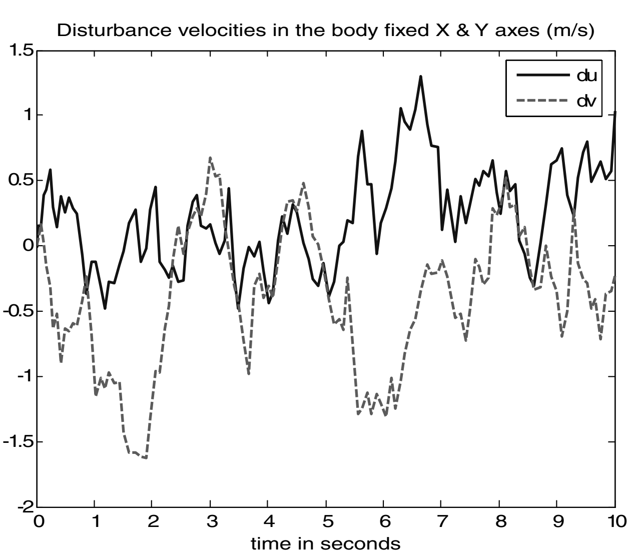

Fig. 6 Random disturbance vector.

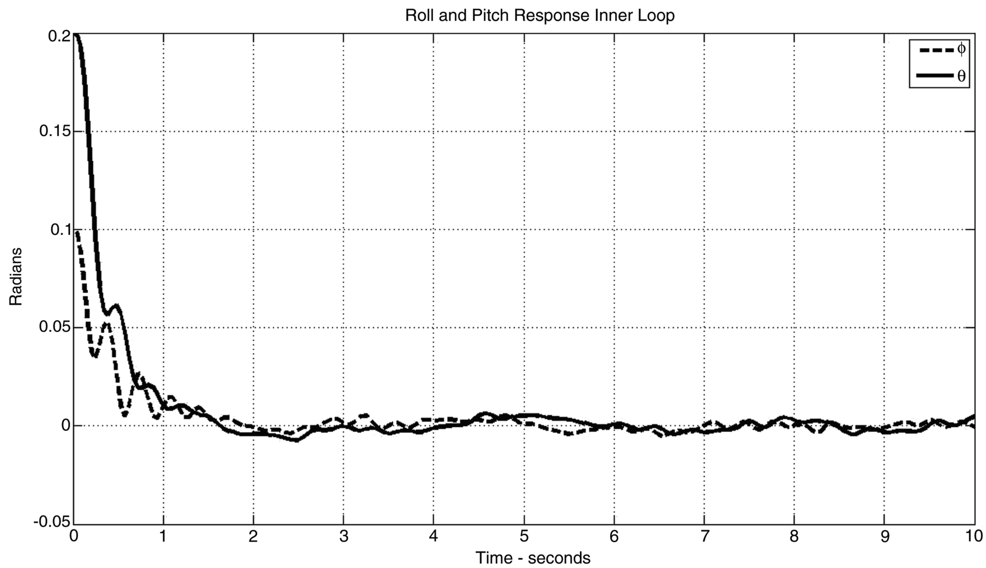

Fig. 7 Roll and pitch responses (inner loop).

position of the UAV in hover. Position-tracking in hover is addressed in the next section.

E. Outer-Loop Controller Design

To meet the objective to maintain the station position, it is imperative to add an outer tracking loop (the primary variables to be controlled in the outer loop are the positions X, Y, and Z; body rates U, V, and W; and yaw angle). Seven system states constitute the outer-loop output vector:

$$y_o = [X \quad Y \quad Z \quad U \quad V \quad W \quad \psi]^T$$

The control structure is shown in Fig. 8. Precompensators $G_{\mathrm{lat}}(s)$, $G_{\mathrm{long}}(s)$, $G_{\mathrm{col}}(s)$, and $G_{\mathrm{ped}}(s)$ shape the commands from the control law before closing the loop. The loop-shaping procedure is explained in Sec. IV.G. In this example, loop-shaping for the outer loop is not required for stationkeeping because

1) Addition of the effective integrators for synthesizing positions from the transformed velocity components (body to inertial) compensates for the band-limited zero-mean disturbance noise getting into the lateral channel.

2) Coupling in the position-control commands is negligible.

Note that, in general, loop-shaping may be required to deal with disturbances that are not exactly the kind discussed in this paper.

The inner closed loop without the disturbance dynamics can be shown to be given by

$$\dot{x}_{\mathrm{in}} = A_{\mathrm{ic}} x_{\mathrm{in}} + B_{\mathrm{in}} u_{\mathrm{ic}} \tag{14}$$

where $A_{\mathrm{ic}} = A_{\mathrm{in}} - B_{\mathrm{in}} K_{\mathrm{in}} C_{\mathrm{in}}$ is the inner closed-loop system matrix. An inertial measurement unit can be used to measure the position [29]. Assuming zero bank and pitch angles, the position and yaw dynamics are

$$\frac{\mathrm{d}}{\mathrm{d}t}[\psi \quad X \quad Y \quad Z]^T = [r \quad U \quad V \quad W]^T \tag{15}$$

The overall state vector, including position and yaw states, can be expressed as

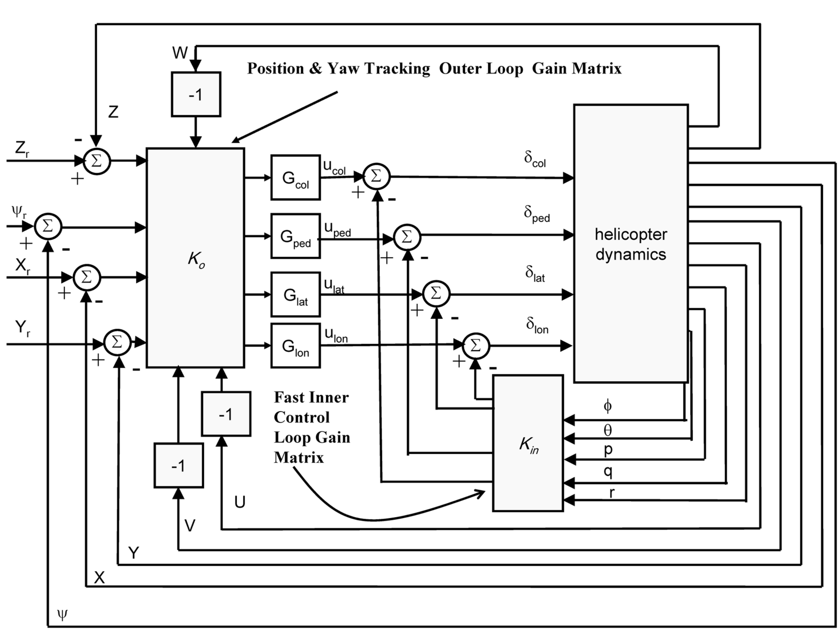

Fig. 8 Outer- and inner-loop controller structure.

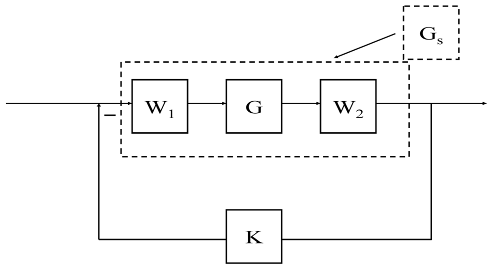

Fig. 9 Loop-shaped plant with controller.

$$x_o = [U \quad V \quad p \quad q \quad \phi \quad \theta \quad a_s \quad b_s \quad W \quad r \quad r_{\mathrm{fb}} \quad \psi \quad X \quad Y \quad Z]^T$$

One can easily find the matrix H of dimension 4×11 that satisfies

$$\frac{\mathrm{d}}{\mathrm{d}t}[\psi \quad X \quad Y \quad Z]^T = Hx_{\mathrm{in}} \tag{16}$$

working with Eqs. (15) and (16) to obtain

$$\dot{x}_o = \left[\frac{A_{\mathrm{ic}}}{H}\middle|\frac{0}{0}\right]x_o + \left[\frac{B_{\mathrm{in}}}{0}\right]u_{\mathrm{ic}}, \qquad y_o = [X \quad Y \quad Z \quad U \quad V \quad W \quad \psi]^T \tag{17}$$

The augmented dynamics including disturbance effects is given as

$$\dot{x}_o = A_o x_o + B_o u_{\mathrm{ic}} + D_o d_o, \qquad y_o = C_o x_o \qquad x_o \in R^{15}, \qquad u_{\mathrm{ic}} \in R^4, \qquad y_o \in R^7 \tag{18}$$

An outer-loop output-feedback control input is selected as

$$u_{\mathrm{ic}} = -K_o v_r \tag{19}$$

where

$$v_r = [(X - X_r) \quad (Y - Y_r) \quad (Z - Z_r) \quad U \quad V \quad W \quad (\psi - \psi_r)]^T \tag{20}$$

Note that the vector v_r facilitates remote pilot-tracking commands (X_r, Y_r, Z_r) being sent, whereas measured output variables y_o are used for feedback onboard. The output-feedback gain K_o is determined from the algorithm described in Sec. III using the state variable model described in Eq. (18). Note that $Q^T = Q \geq 0$ and $R^T = R > 0$. The system matrices A_o and B_o can be easily found by simple algebraic manipulations, and output matrix C_o is composed of unity and zero elements so that outer-loop measurement states are included. The outer-loop disturbance dynamics are discussed in Sec. IV.F. In the outer loop, the weighting matrices Q and R are chosen such that position-tracking states are weighted more and the control inputs are within the specified range of actuation [30]. The gain parameter γ_o in the outer loop is found to be 1.3450. The resulting outer-loop output-feedback gain is

$$K_o = \begin{bmatrix} 0.0250 & 0.2808 & 0.0206 & 0.0355 & 0.3426 & 0.0110 & -0.0196 \\ -0.2554 & 0.0204 & 0.0320 & -0.3351 & 0.0152 & 0.0166 & -0.0266 \\ 0.0302 & -0.0214 & 0.2555 & 0.0452 & -0.0286 & 0.1652 & 0.0680 \\ 0.0023 & -0.0018 & 0.0044 & 0.0033 & -0.0026 & 0.0049 & -3.6507 \end{bmatrix}$$

F. Disturbance Effects in the Outer Loop

In the inner loop, we used the model of the wind components as disturbance. To further see the efficacy of the design, disturbances can be formulated by injecting noise into the helicopter control channels. In this particular example, the disturbance is injected into the lateral channel via an external wind gust affecting the lateral channel. The disturbance input matrix in the outer loop is given as

$$D_o = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0.0632 \quad 3.1739 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

A band-limited white-noise source is used to simulate disturbance input signal d_o.

G. H_∞ Loop-Shaping Design Procedure

We will now formally state the design procedure for loop-shaping. The loop-shaped plant with controller is shown in Fig. 9. The objective of this approach is to balance the tradeoff between performance and robustness in loop-shaping. The procedure couples loop-shaping design with H_∞ output-feedback control techniques. There exists a large amount of published material relating to loop-shaping [32], including using a precompensator W_1 and a postcompensator W_2 to shape the singular values of the nominal plant to achieve a desired open loop, combining the nominal plant G and the compensators to form the shaped plant G_s, choosing weighting matrices Q and R for G_s, and using the H_∞ static output-feedback algorithm to find the static output-feedback gain. The algorithm is described in Sec. III.

H. Outer-Loop Simulation Results with Disturbance Effects

The figures described here illustrate the effectiveness of the design. Case 1 in Fig. 10 shows the tracking plots for position commands. The UAV is commanded from initial position vectors (X_1, Y_1, Z_1) to (X_2, Y_1, Z_1), (X_1, Y_1, Z_1) to (X_1, Y_2, Z_1), and (X_1, Y_1, Z_1) to (X_1, Y_1, Z_2). The results clearly show command-following as well as channel-decoupling.

Case 2 in Fig. 11 shows the regulation properties of the closed-loop system: that is, how the UAV converges to the assigned station position (X, Y, Z) from a different initial position vector ($X + \Delta X$, $Y + \Delta Y$, and $Z + \Delta Z$). One can also see that the yaw angle is maintained at the commanded value. Case 3 in Fig. 12 is an illustration of the successful following of a yaw step command while maintaining the position. Figure 13 shows the disturbance rejection of a disturbance in the lateral channel.

Remark: The net result is that the inner attitude-control loop is faster than the outer position-control loop, and so the attitudes respond quickly to allow effective position-tracking. This is clearly seen by comparing Figs. 7 and 12 with 10. . The response times T_r for the position states X, Y, and Z are 2.7387, 2.5041, and 1.3117 s, and the response times T_r for attitude states ϕ, θ, and ψ are 0.4960, 0.5612, and 0.1797 s, where parameter T_r is the time taken from 10 to 90% of the final value [33].

One can impose a time-scale separation by selecting the LQR state weighting matrix to be larger in the inner-loop design. In this example, the design has actually automatically imposed a time-scale separation.

In some cases, when relative LQR weighting is not satisfactory, one can further enforce the time-scale separation by loop-shaping with the filters. Note that the framework is general enough to include precompensators and postcompensators so that classical loop-shaping can be applied to improve the performance, followed by a rigorous H_∞ solution given in [11]. A set of constant gains achieves these control tasks, and the order of the controller is the lowest possible.

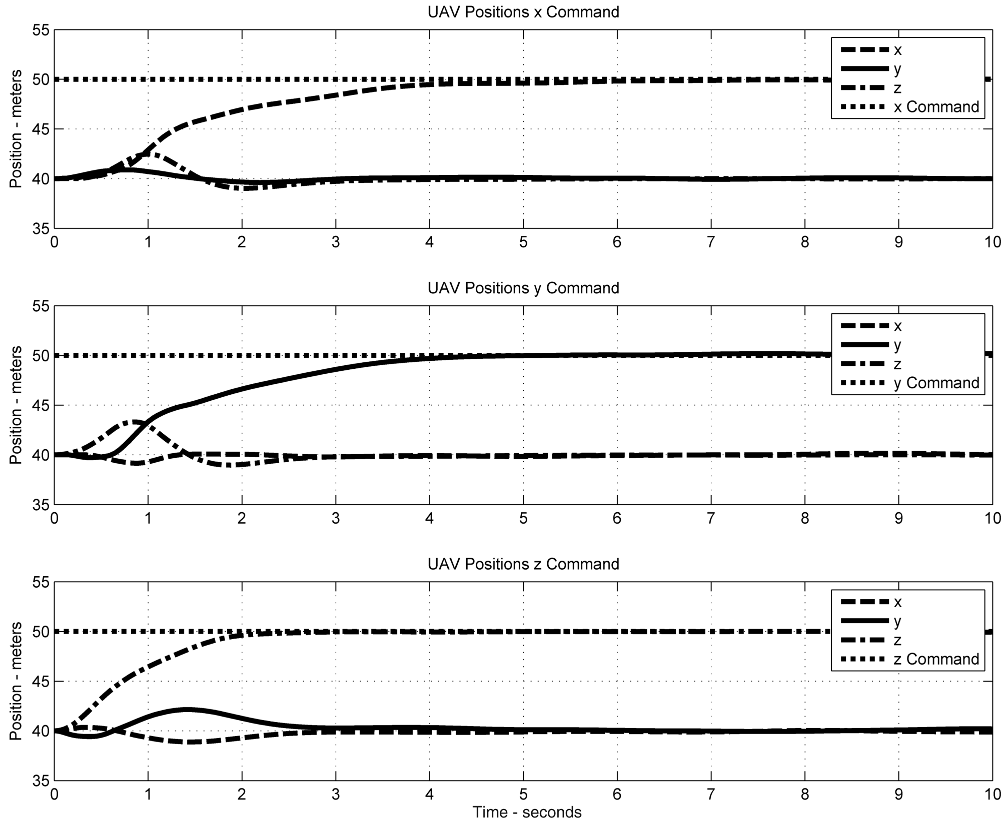

Fig. 10 Case 1: position commands.

V. Conclusions

The problem of disturbance attenuation with stability using static output feedback for linear time-invariant systems was studied. Necessary and sufficient conditions were developed, which yield two coupled-matrix design equations to be solved for the output-feedback (OPFB) gain. A computational algorithm to solve for the output-feedback gain that achieves prespecified disturbance attenuation is given. The algorithm requires no initial stabilizing

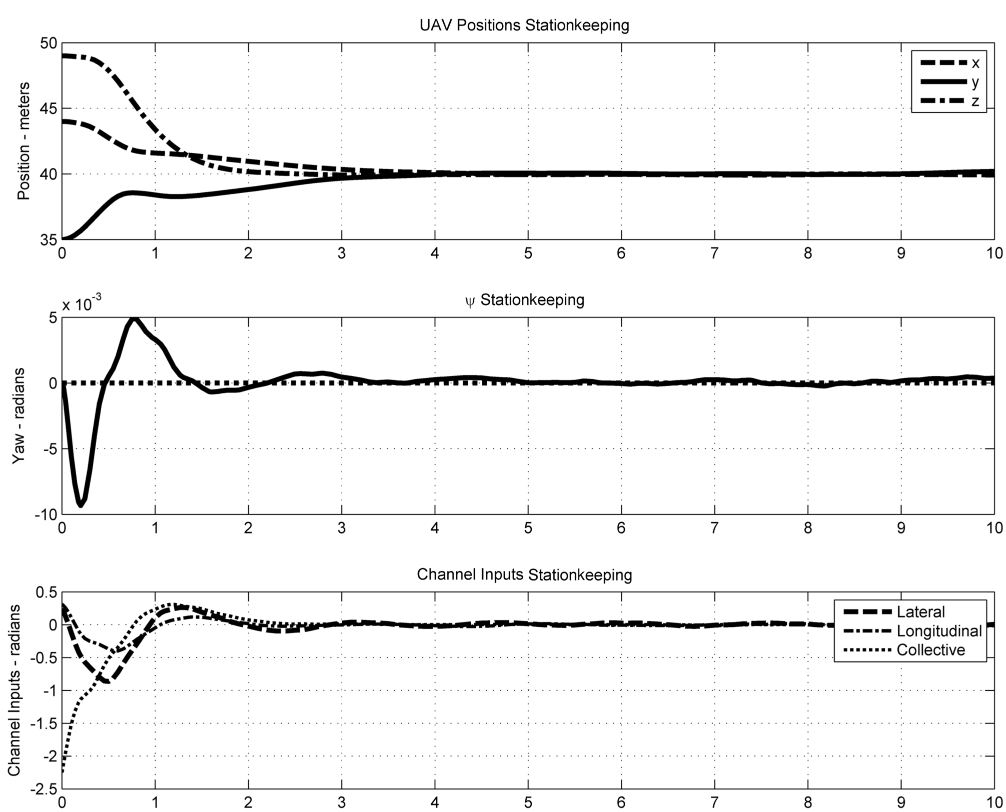

Fig. 11 Case 2: stationkeeping.

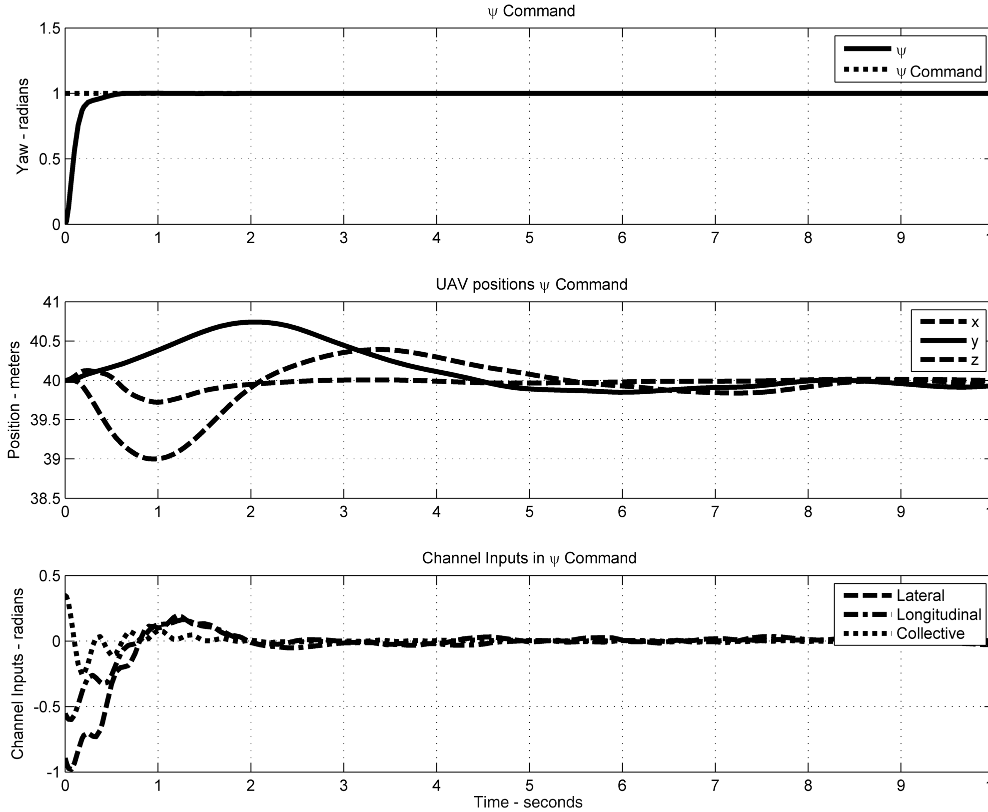

Fig. 12 Case 3: yaw tracking while maintaining position.

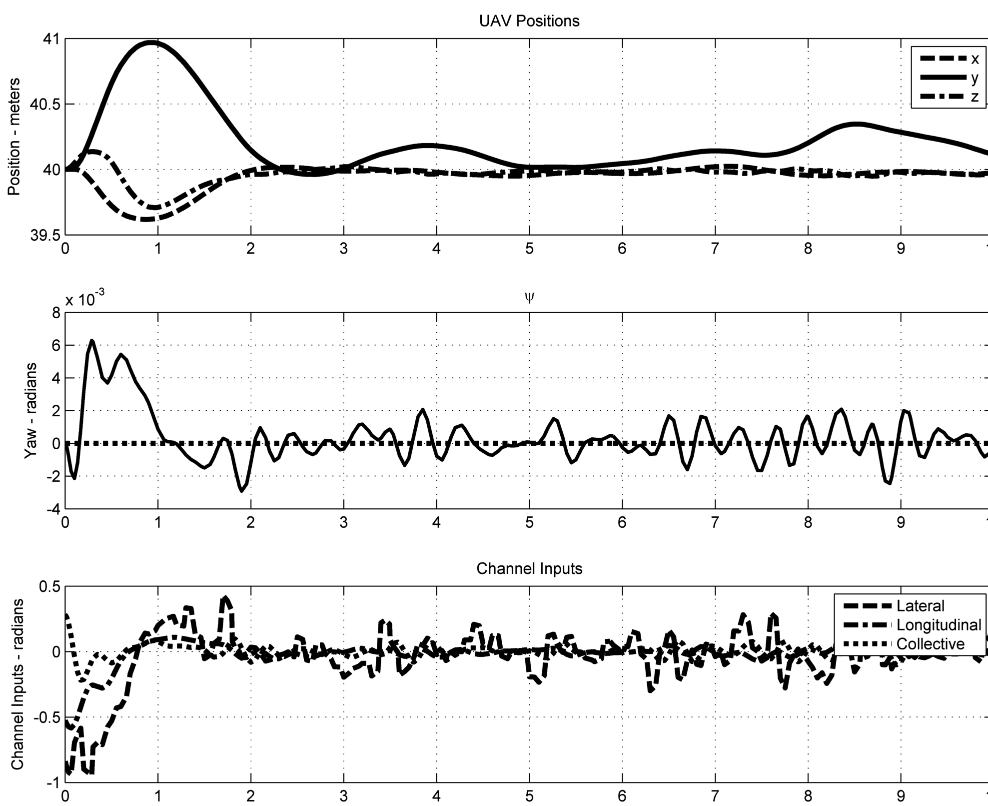

Fig. 13 Case 4: lateral wind gust.

gain, in contrast to other existing recursive OPFB solution algorithms. This procedure allows output-feedback control design with prespecified controller structures and guaranteed performance. A robust controller for stabilizing an autonomous rotorcraft in hover was designed using the algorithm defined in the paper.

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